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Modal Characteristics in NRD and H-Guides with Lossy Dielectric Strips

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The modal characteristics of non-radiative dielectric (NRD) and H-guides with lossy dielectric strips are analyzed using complex propagation constants that are rigorously obtained using Davidenko's complex root-finding algorithm. For the dominant mode of the lossy NRD guide, there is no cutoff frequency, and this mode is no longer a trapped surface wave (TSW) due to dielectric loss. The TSW mode is changed to surface-wave-like (SWL) mode, and its effective cutoff frequency is defined. For the first higher-order mode of the lossy NRD guide, the cutoff frequency in dispersion curve is not the actual cutoff frequency because it resides in the reactive region. Thus, an effective cutoff frequency can also be defined for this mode. This mode is also a SWL mode. For the general higher-order modes of a lossy NRD guide, as dielectric loss increases, the spectral gap, which is known to be a nonphysical region in the lossless case, becomes narrower and changes to physical regions, such as forward and backward leaky wave regions. If the dielectric strip is sufficiently lossy, eventually, the spectral gap disappears. Even the TSW region also changes to a backward leaky wave region in a specific frequency range.

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I. INTRODUCTION

Millimeter waves are electromagnetic waves whose wavelengths are 1.0 to 10.0 mm [1], which have been extensively used in passive [2] and active [3, 4] devices. Non-radiative dielectric (NRD) and H-guides have been widely utilized as low-loss transmission lines in millimeter-wave frequencies. They have a dielectric strip sandwiched by parallel plate waveguides (PPWs) and have the same geometry except for the distance between plates. Their modal characteristics have been extensively studied with a lossless dielectric strip [5]. However, the complex modal characteristics should be investigated because all practical dielectric materials are lossy.

In this research, we investigated the dispersion characteristics of an NRD guide with a lossy dielectric strip. In order to obtain rigorous complex propagation constants, we employed Davidenko's method [6]. When the dielectric strip is lossy, the solution of each mode from the characteristic equation leads to complicated dispersion curve, and the solution represents various mode types, such as proper complex solutions, that is, backward waves, backward leaky waves, and surface-wave-like (SWL) modes and improper complex solutions, that is, spectral gaps, forward leaky waves, and SWL modes. Especially, backward waves, backward leaky waves, and SWL modes are found only for lossy strip; they are not found for a lossless strip. In addition, the effective cutoff frequencies of the dominant and the first higher-order modes are defined. The single-mode operating frequency range of a lossy NRD guide is also defined.

II. COMPLEX CHARACTERISTIC EQUATION AND PROPAGATION CONSTANTS

A cross section of an NRD guide with a lossy dielectric strip is shown in Fig. 1. If an NRD guide is to be operated with a single dominant mode of LSM_{01} at 12.5 GHz, the height h and the width $2W$ must be set at 10.8 and 9.7 mm ($h/\lambda_0 = 0.45,\ 2W\sqrt{\varepsilon_r - 1}/\lambda_0 = 0.5$), respectively. The dielectric constant of a strip is assumed to be complex. Then, the complex characteristic equation with a lossy strip can be obtained using the boundary condition of tangential field and is given by

$$F = \hat{k}_{x0} \pm j\frac{\hat{k}_{x\varepsilon}}{p}\left\{\begin{array}{c}\tan\hat{k}_{x\varepsilon}\\ \cot\hat{k}_{x\varepsilon}\end{array}\right\} = 0, \qquad \begin{array}{l}\text{even mode}\\ \text{odd mode}\end{array} \quad (1)$$

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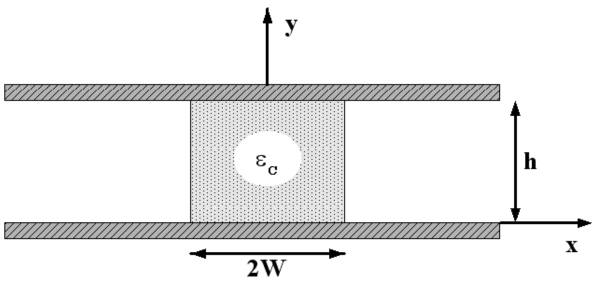

Fig. 1. Cross-sectional view of the NRD and the H-guide structures.

where p is defined as $p = \varepsilon_c$ and $p = 1$ for the longitudinal section magnetic (LSM) and the longitudinal section electric (LSE) modes, respectively, and $\varepsilon_c = \varepsilon_r(1 - j\tan\delta)$, $\hat{k}_{x0} = k_{x0}W$, and $\hat{k}_{x\varepsilon} = k_{x\varepsilon}W$. The relative permittivity ε_r is 2.53, and the loss tangent $\tan\delta$ is set to 0.001 or 0.1, which corresponds to a low or a high dielectric loss. Also, k_{x0} and $k_{x\varepsilon}$ are the respective transverse wave-numbers in air and in the dielectric region and can be expressed as $k_{x0} = \beta_{x0} - j\alpha_{x0}$ and $k_{x\varepsilon} = \beta_{x\varepsilon} - j\alpha_{x\varepsilon}$. $\beta_{x0}(\beta_{x\varepsilon})$ and $\alpha_{x0}(\alpha_{x\varepsilon})$ are the phase and the attenuation constant of the transverse direction in air (dielectric), respectively.

In air and in the dielectric, the dispersion relations of the longitudinal and the transverse directions are given by

$$k_z^2 = \begin{cases} k_0^2 - \left(\frac{m\pi}{h}\right)^2 - k_{x0}^2, & \text{air} \\ k_0^2\varepsilon_c - \left(\frac{m\pi}{h}\right)^2 - k_{x\varepsilon}^2, & \text{dielectric} \end{cases} \tag{2}$$

Here, $k_z = \beta_z - j\alpha_z$ is a complex propagation constant for the longitudinal direction, where β_z and α_z are the phase and the attenuation constant, respectively. The propagation constants satisfying Eq. (1) are rigorously obtained using Davidenko's method. The main equation of Davidenko's method can be expressed two equations, which are two coupled, nonlinear, first-order ordinary differential equations with a dummy variable of t:

$$\begin{cases} \frac{d\bar{\beta}_z}{dt} = -\frac{Re[F]Re[F_{kz}]+Im[F]Im[F_{kz}]}{|F_{kz}|^2} \\ \frac{d\bar{\alpha}_z}{dt} = -\frac{Re[F]Im[F_{kz}]-Im[F]Re[F_{kz}]}{|F_{kz}|^2} \end{cases} \tag{3}$$

where $\bar{\beta}_z = \beta_z/k_0$ and $\bar{\alpha}_z = \alpha_z/k_0$. k_0 is the free space wave-number. F_{kz} is the partial derivative of F with respect to k_z, that is, $F_{kz} = dF/dk_z$. The solutions of Eq. (3) can be reached to a desired precision when t is set to be very large [6]. The modes of the NRD guide consist of LSM and LSE modes. Of these modes, the dominant LSE mode is considered to be parasitic because its H-field component is predominantly parallel to the plates, thereby generating a conduction current and increasing conduction loss as the frequency is increased. Consequently, the total loss increases with frequency. Thus, the LSM mode is the desirable operating mode. This mode has its E-field component predominantly parallel to the plates and, hence, forms a low-loss propagating mode. Thus, we considered only LSM modes in this paper.

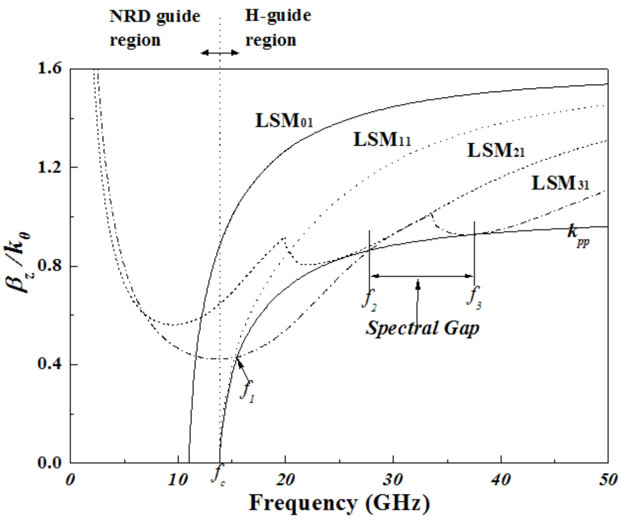

Fig. 2. In the lossless case, the dispersion characteristics in NRD and H-guides.

III. NUMERICAL RESULTS

For comparison of all modes, at first, we considered a lossless case. As Fig. 2 shows, the dominant LSM_{01} and first higher-order LSM_{11} modes have cutoff frequencies of 11.06 and 13.89 GHz, respectively. Their propagation constants are purely real (propagation) or purely imaginary (attenuation). Thus, the modal solutions are proper and real, which is similar to the case of a typical closed structure. Thus, these modes turned out to be trapped surface waves (TSWs) ($\alpha_z, \beta_{x0} = 0$) above the cutoff frequencies. The second (LSM_{21}) and third (LSM_{31}) higher-order modes have no cutoff frequencies and have various type of modes. There are nonphysical reactive modes, TSW modes, forward leaky modes, and spectral gaps. The spectral gap connects a leaky wave and a TSW as shown in Fig. 2 [5]. In Fig. 2, k_{pp} is the dispersion curve of the PPW, and it judges whether the guided wave is slow or fast in the NRD and H-guides. In other words, if $k_{pp} > \bar{\beta}_z$, the waves are fast waves, such as forward and backward leaky modes. If $k_{pp} < \bar{\beta}_z$, the waves are slow waves, such as TSWs, backward SWLs, and forward SWL modes.

1. LSM_{01} Mode

When a dielectric strip is lossy, the dominant mode has no cutoff frequency, as shown in Fig. 3, and the propaga-

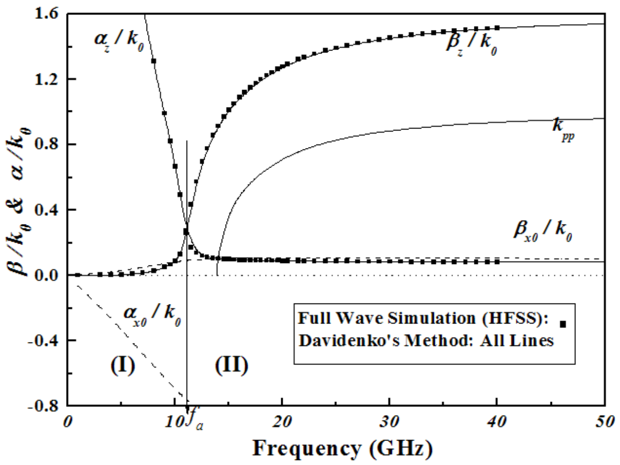

Fig. 3. In the lossy case, the dispersion characteristics of the dominant mode in NRD and H-guides ($\tan\delta = 0.1$, (I): reactive mode region and (II): SWL mode region).

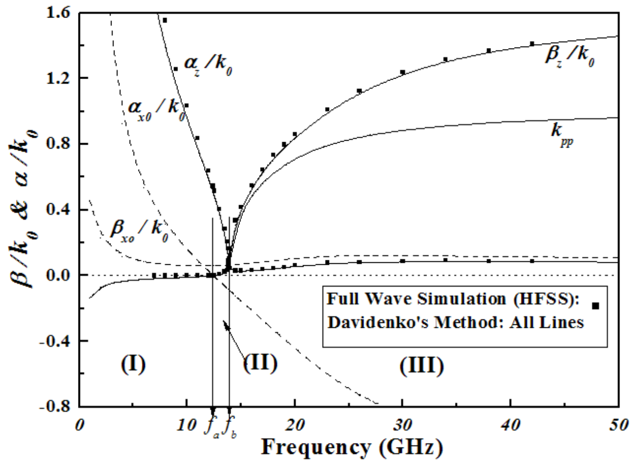

Fig. 4. In the lossy case, the dispersion characteristics of the first higher-order mode in NRD and H-guides ($\tan\delta = 0.1$, (I): proper complex, reactive mode region, (II): improper complex, reactive mode region, and (III): SWL mode region).

tion constants of the longitudinal (solid line: β_z, α_z) and the transverse (dashed line: β_{x0}, α_{x0}) directions become complex. To validate thee solutions obtained from the complex characteristic equation by using Davidenko's method, we used a finite element method (FEM) to compare them with the simulation results over the whole frequency range. As Fig. 3 shows, the results from Davidenko's method agree well with the FEM simulation. The modal solutions shown in Fig. 3 are improper (nonspectral) since their signs are $\beta_z > 0, \alpha_z > 0, \beta_{x0} > 0$ and $\alpha_{x0} < 0$. This complex wave corresponds to a forward leaky, but slow, wave with the condition of $k_{pp} < \beta_z$ over the whole frequency range, as shown in Fig. 3 [7]. Thus, the LSM_{01} mode is no longer a TSW because α_z and β_{x0} are not zero. If the modal solution is like an improper complex and slow wave, then we can call this wave a "surface-wave-like mode" because its field distribution resembles that of a TSW in an NRD guide with a lossless dielectric strip [8,9].

Although the modal solution is like a SWL mode over the whole frequency, it is characterized by two regions bounded by f_a (11.14 GHz), where α_z is equal to β_z. First, below f_a in Fig. 3, the modal solution corresponds to improper complex solutions, a SWL mode with $\beta_z << \alpha_z$ (reactive mode) and $\beta_{x0} << |\alpha_{x0}|$. This wave represents a rapidly decaying wave both in the longitudinal and the transverse directions. Thus, in this frequency range, the mode does not contribute to the total fields as in the reactive wave region [10]. Second, above f_a, the solution is improper complex with $\beta_z >> \alpha_z$ (radiating mode). Then, the wave propagates along the dielectric surface in the longitudinal direction and, at the same time, radiates into the air region. However, the wave radiating into air is suppressed in the transverse direction because of the cutoff nature of the PPW. Thus, the wave type of this region propagates in the longitudinal direction in a SWL mode. Even though no cutoff frequency is found for the LSM_{01} mode in Fig. 3, an effective cutoff frequency can be defined by using the critical point f_a that separate the solution into two regions, reactive and propagating.

2. LSM_{11} Mode

Fig. 4 shows the dispersion and the attenuation curves of LSM_{11} mode. When the dielectric strip is lossy, the LSM_{11} mode can be divided into the three regions bounded by f_a (12.38 GHz) and f_b (13.89 GHz) according to modal solution type shown in Fig. 4. f_a and f_b are points with $\beta_z = 0$ and $\beta_z = \alpha_z$, respectively. In region I, below f_a, since the signs of the propagation constants are $\beta_z < 0, \alpha_z > 0, \beta_{x0} > 0$ and $\alpha_{x0} > 0$, the modal solutions are proper (spectral) complex, which corresponds to a backward wave solution with $\beta_z << \alpha_z$ [7]. Thus, this wave is directed in a direction opposite to the longitudinal direction and its power is stored at the port. That is, this region is a reactive mode region as with a backward wave. In region II, from f_a to f_b, with $\beta_z > 0, \alpha_z > 0, \beta_{x0} > 0$ and $\alpha_{x0} < 0$, the modal solutions are improper complex with $\beta_z << \alpha_z$. Thus, this region is also a reactive mode region as with a forward wave. In region III, above f_b, since the signs of the propagation constants are $\beta_z > 0, \alpha_z > 0, \beta_{x0} > 0$ and $\alpha_{x0} < 0$, the modal solutions are improper complex and correspond to a forward leaky, but slow wave solution with $\beta_z >> \alpha_z$. A wave in this region has the same form as the LSM_{01} mode in region II. Thus, region III is the SWL mode region. Graphically, the cutoff frequency of the LSM_{11} mode is f_a, which is located in the reactive region. Thus, the effective cutoff frequency is f_b, above which the wave propagates. In the lossy case, between f_a

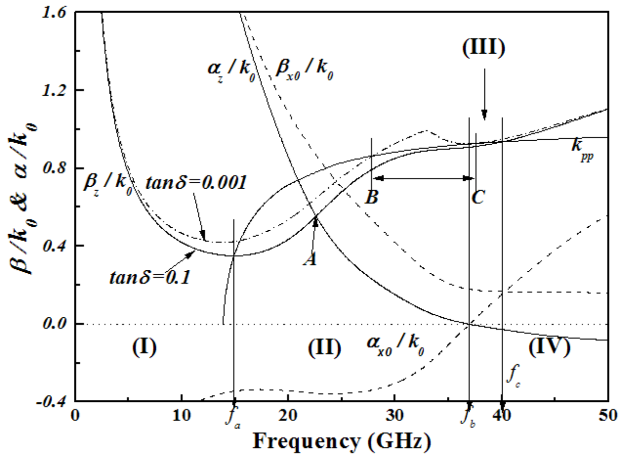

Fig. 5. In the lossy case, the dispersion characteristics of the general higher-order mode in NRD and H-guides ($\tan\delta$ = 0.001 and 0.1, (I): nonphysical reactive mode region, (II): forward leaky wave mode region, (III): backward leaky wave mode region, and (IV): SWL mode region).

in Fig. 3 and f_b in Fig. 4, therefore, is the single-mode operating region for an NRD guide with a lossy dielectric strip.

3. LSM$_{31}$ Mode

As Fig. 2 shows, the values of the β_z/k_0 of the LSM$_{21}$ mode are always greater than k_{pp}, which means that the LSM$_{21}$ mode is not a leaky mode at any frequency. Thus, a transition region between the nonphysical and the physical regions does not exist, and the dispersion curve of the LSM$_{21}$ mode only consists of a nonphysical region and a physical region (TSW mode). In that sense, the LSM$_{21}$ mode cannot be regarded as a general higher-order mode.

Here, we concentrate on the analysis of the LSM$_{31}$ mode as a general higher-order mode. Fig. 5 shows the dispersion curve of the LSM$_{31}$ mode for a general higher-order mode. The dash-dot and the solid lines show the phase constants of the mode in longitudinal direction when the loss-tangent value is 0.001 and 0.1, respectively. When the dielectric loss is 0.001, the spectral gap is narrower than it is in the lossless case. As Fig. 2 and Fig. 5 show, the spectral gaps of the lossless and the lossy dielectric strips inserted are 27.80 to 37.58 GHz and 27.87 to 37.27 GHz, respectively. Thereby, we find that the spectral gap of the nonphysical region changes to the physical leaky region when dielectric loss exists. Here, it is also expected that if the dielectric loss increases, the spectral gap will disappear because increasing loss makes the spectral gap region gradually narrower.

In Fig. 5, the region between B and C (27.80 – 37.58 GHz) is occupied with the spectral gap in the case of a lossless dielectric strip, as shown in Fig. 1. However,

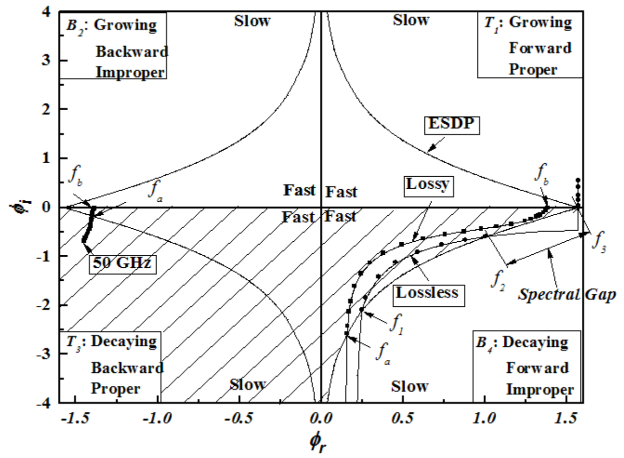

Fig. 6. Behavior of pole solutions of the LSM$_{31}$ mode in Figs. 2 and 5 on the ϕ-plane ($\tan\delta$ = 0 and 0.1).

there is no spectral gap in the LSM$_{31}$ mode in Fig. 5 when the dielectric strip has sufficient loss, *i.e.*, $\tan\delta$ = 0.1. That is, the nonphysical spectral gap region has changed to physical forward (B – f_b) and backward (f_b – C) leaky wave regions, as shown in Fig. 5. The region between C and f_c (40.04 GHz) is occupied with a TSW in the case of a lossless dielectric strip. In this region, the TSW region has changed to a physical backward leaky region. Each region can be categorized in the same manner as previously mentioned. Thus, regions I, II, III, and IV correspond to nonphysical reactive, physical forward leaky, physical backward leaky, and SWL modes, respectively. Especially, region II from f_a to f_b is divided into two distinct regions bounded by the point A (22.5GHz). At this point with $\alpha_z = \beta_z$, the region from f_a to A corresponds to a reactive mode region ($\alpha_z > \beta_z$) and the region from A to f_b is an antenna mode region ($\alpha_z < \beta_z$).

4. Steepest Decent Plane (SDP) Analysis

It can be judged whether the complex solution of the LSM$_{31}$ mode is physical or not from its behavior on the steepest decent plane (SDP). If the pole solution of k_z is physical, the pole should be captured into physical regions on the ϕ-plane when the original path is deformed into the extreme steepest descent path. In an NRD guide, the SDP curves in the ϕ-plane are defined from $k_z = k_{pp}\sin\phi$, where $\phi = \phi_r + i\phi_i$. The k_z plane can be mapped onto the ϕ-plane by using this relation. Thus, the phase and the attenuation constants can be transformed onto the ϕ-plane, respectively, by using

$$\begin{bmatrix} \beta_z = k_{pp}\sin\phi_r\cosh\phi_i \\ -\alpha_z = k_{pp}\cos\phi_r\sinh\phi_i \end{bmatrix} \tag{4}$$

In Fig. 6, the extreme steepest descent path is defined when β_z is equal to k_{pp} in Eqs. (4), *i.e.*, when

Table 1. Classification of the LSM_{31} mode in a lossy NRD guide.

Frequency range, sheet	Property of each range	Classification of LSM_{31} mode
$f_a > f$, B_4 sheet	Decaying, forward, improper and slow wave	Nonphysical reactive wave mode
$f_a < f < f_b$, B_4 sheet	Decaying, forward, improper and fast wave	Forward leaky wave mode
$f_b < f < f_c$, T_3 sheet	Decaying, backward, proper and fast wave	Backward leaky wave mode
$f > f_c$, T_3 sheet	Decaying, backward, proper and slow wave	SWL mode

$\sin\phi_r \cosh\phi_i = 1$, which is the boundary of the fast and slow wave and is also used to determine whether the solutions are physical or not [5]. To assist with the physical interpretation of each sheet, we labeled each sheet as T_1, B_2, T_3, and B_4 in the ϕ-plane. Here, each labeling indicates whether the solution is decaying or growing in the longitudinal direction, a forward or backward wave, or a proper or improper wave in the transverse direction. The notations B and T mean the bottom (improper) and the top (proper) sheets of the Riemann surfaces about the k_z plane. Therefore, the sheets T_1 and B_2 are not physical because a growing wave is nonphysical. The sheet T_3 is physical because physical backward waves are proper, regardless of whether they are fast or slow. The sheet B_4 is physical only in the fast wave region because that region can support a forward leaky wave as an improper wave [11]. Thus, in Fig. 6, the shaded regions are physical, and the other regions are nonphysical.

Fig. 6 shows the SDP curves on the ϕ-plane with respect to the pole solutions of the LSM_{31} in Fig. 2 and Fig. 5. In Fig. 6, the solid line with quadrilateral symbols and the solid lines with circular symbols are for lossy and lossless strips, respectively, and represent only the physical frequency ranges of the LSM_{31} modes. The specific frequency points for the LSM_{31} mode in Figs. 2 and 5 are exactly identical to those in Fig. 6. Therefore, the LSM_{31} mode in a certain frequency range is known to be either physical or nonphysical in Figs. 2 and 5 depending on whether or not the poles are captured when the original path is deformed into the extreme steepest descent path in Fig. 6. That is, in Fig. 6, below f_a, the pole solutions of the LSM_{31} mode pass through the slow wave region. Between f_a and f_b, the pole solutions of the LSM_{31} mode are captured by the extreme steepest descent path in the fast wave region, a physical region. Between f_b and f_c, the pole solutions are captured in the fast wave region of the T_3 sheet, and above f_c, the pole solutions are captured in the slow wave region of the T_3 sheet. Finally, the LSM_{31} mode of a lossy NRD guide, Fig. 6 can be summarized as listed in Table 1. Therefore, the results of the SDP analysis are exactly identical to those of a complex wave analysis and the spectral gap disappears in a lossy strip, as shown in Fig. 6.

IV. CONCLUSIONS

Modal analyses of NRD and H-guides with lossy dielectric strips were made by using the complex propagation constants that were obtained by using Davidenko's method. When the dielectric strip is lossy, the modal solutions are very different from those in lossless case. In particular, there is no cutoff frequency in the dispersion curve, and the TSW changes to a SWL mode for the dominant mode. The first higher-order modes also turn out to be a SWL mode, and its effective cutoff frequency is defined. Thus, the single-mode operating frequency is defined by using these effective cutoff frequencies of the dominant and the first higher-order modes. For general higher-order modes, the spectral gap region, which is nonphysical, gradually evolves to physical region, such as a backward or a forward leaky wave region. Also, the TSW region changes to a backward leaky wave region in a specific frequency range. Especially, when the dielectric strip is sufficiently lossy, the spectral gap disappears.

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